5.4.1. Metoda parcijalne integracije za neodređene integrale

18. 12. 2020.

Fotografija: Roma Kaiuk

Parcijalna integracija za neodređene integrale

Neka su $u, v : I \subseteq \mathbb{R} \to \mathbb{R}$ derivabilne funkcije. Imamo

$$(uv)' = u'v + uv',$$

tj.

$$uv' = (uv)' - u'v,$$

odakle integriranjem dobivamo **formulu parcijalne integracije**

$$\int uv' = uv - \int u'v.$$

Popularno je (i korisno u računu) zapisivati je kratko i neprecizno u obliku

$$\boxed{\int u \,\mathrm{d}v = uv - \int v \,\mathrm{d}u.}$$

Primjer 1

Koristeći formulu parcijalne integracije

$$\boxed{\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u,}$$

izračunajmo integral

$$\int xe^x\,\mathrm{d}x.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

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Imamo

$$\int xe^x\,\mathrm{d}x = \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^x\,\mathrm{d}x & v = e^x \end{bmatrix}$$

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Zadatak 49(a)

Izračunajte integral $\int x \sin(6x)\,\mathrm{d}x$.

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Rješenje. Imamo

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$$\begin{aligned}\int x \sin(6x)\,\mathrm{d}x &= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = \sin(6x)\,\mathrm{d}x & v = -\frac{1}{6}\cos(6x)\end{bmatrix} \\ &= -\frac{1}{6}x\cos(6x) + \frac{1}{6}\int \cos(6x)\,\mathrm{d}x\end{aligned}$$

$$\begin{aligned}\int \sin(6x)\,\mathrm{d}x = \begin{bmatrix} t = 6x \\ \mathrm{d}t = 6\,\mathrm{d}x\end{bmatrix} = \int \sin t \cdot \frac{\mathrm{d}t}{6} = -\frac{1}{6}\cos t + C \\ = -\frac{1}{6}\cos(6x) + C.\end{aligned}$$

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$$= -\frac{1}{6}x\cos(6x) + \frac{1}{6}\int \cos(6x)\,\mathrm{d}x$$

$$\overset{\text{sami}}{=} -\frac{1}{6}x\cos(6x) + \frac{1}{36}\int \sin(6x) + C,$$

pri čemu funkciju v odredimo sljedećim računom:

$$\int \sin(6x)\,\mathrm{d}x = \begin{bmatrix} t = 6x \\ \mathrm{d}t = 6\,\mathrm{d}x \end{bmatrix} = \int \sin t \cdot \frac{\mathrm{d}t}{6} = -\frac{1}{6}\cos t + C$$

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Jako korisna napomena

Ako je

$$\int f(x)\,\mathrm{d}x = F(x) + C$$

(dakle $F' = f$), tada je za sve $a \in \mathbb{R} \setminus \{0\}$ i $b \in \mathbb{R}$

$$\boxed{\int f(ax+b)\,\mathrm{d}x = \frac{1}{a}F(ax+b) + C.}$$

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Dokaz. $\left(\frac{1}{a}F(ax+b)\right)' = \frac{1}{a}F'(ax+b)\cdot a$

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Dokaz. $\left(\frac{1}{a}F(ax+b)\right)' = \frac{1}{\cancel{a}}F'(ax+b)\cdot \cancel{a}$

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Dokaz. $\left(\frac{1}{a}F(ax+b)\right)' = \frac{1}{\not{a}}F'(ax+b)\cdot \not{a} = f(ax+b).$

Zadatak 49(b)

Izračunajte integral $\int x^2 e^{4x}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$

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$$\int x^2 e^{4x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

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$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int x e^{4x}\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(b)

Izračunajte integral $\int x^2 e^{4x}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2 e^{4x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int x e^{4x}\,\mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(b)

Izračunajte integral $\int x^2 e^{4x}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2 e^{4x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int x e^{4x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

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$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(b)

Izračunajte integral $\int x^2 e^{4x}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2 e^{4x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int xe^{4x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left(\frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}\,\mathrm{d}x\right)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

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$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(b)

Izračunajte integral $\int x^2 e^{4x}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2 e^{4x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int xe^{4x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\left(\frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}\,\mathrm{d}x\right)$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{8}\cdot\frac{1}{4}e^{4x} + C$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(b)

Izračunajte integral $\int x^2 e^{4x}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2 e^{4x}\,\mathrm{d}x = \begin{bmatrix} u = x^2 & \mathrm{d}u = 2x\,\mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int xe^{4x}\,\mathrm{d}x$$

$$= \begin{bmatrix} u = x & \mathrm{d}u = \mathrm{d}x \\ \mathrm{d}v = e^{4x}\,\mathrm{d}x & v = \frac{1}{4}e^{4x} \end{bmatrix}$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left(\frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}\,\mathrm{d}x\right)$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{8}\cdot\frac{1}{4}e^{4x} + C$$

$$= e^{4x}\left(\frac{1}{4}x^2 - \frac{1}{8}x + \frac{1}{32}\right) + C.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \, \mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \, \mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln |x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \, \mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x$$

$$\int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x = \begin{bmatrix} t = 1 - x^2 \\ \mathrm{d}t = -2x \, \mathrm{d}x \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \; a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \,\mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \,\mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \,\mathrm{d}x$$

$$\int \frac{x}{\sqrt{1-x^2}} \,\mathrm{d}x = \begin{bmatrix} t = 1 - x^2 \\ \mathrm{d}t = -2x \,\mathrm{d}x \end{bmatrix} = \int \frac{1}{\sqrt{t}} \cdot \left(-\frac{\mathrm{d}t}{2}\right)$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \, \mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x$$

$$\int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x = \begin{bmatrix} t = 1 - x^2 \\ \mathrm{d}t = -2x \, \mathrm{d}x \end{bmatrix} = \int \frac{1}{\sqrt{t}} \cdot \left(-\frac{\mathrm{d}t}{2}\right)$$

$$= -\frac{1}{2} \cdot 2\sqrt{t} + C_1$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\displaystyle\int \arcsin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \, \mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x$$

$$\int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x = \begin{bmatrix} t = 1 - x^2 \\ \mathrm{d}t = -2x \, \mathrm{d}x \end{bmatrix} = \int \frac{1}{\sqrt{t}} \cdot \left(-\frac{\mathrm{d}t}{2} \right)$$

$$= -\frac{1}{2} \cdot 2\sqrt{t} + C_1 = -\sqrt{1-x^2} + C_1.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(c)

Izračunajte integral $\int \arcsin x \, \mathrm{d}x$.

Rješenje. Imamo

$$\int \arcsin x \, \mathrm{d}x = \begin{bmatrix} u = \arcsin x & \mathrm{d}u = \frac{\mathrm{d}x}{\sqrt{1-x^2}} \\ \mathrm{d}v = \mathrm{d}x & v = x \end{bmatrix}$$

$$= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x$$

$$= x \arcsin x + \sqrt{1-x^2} + C,$$

pri čemu zadnja jednakost vrijedi jer je

$$\int \frac{x}{\sqrt{1-x^2}} \, \mathrm{d}x = \begin{bmatrix} t = 1 - x^2 \\ \mathrm{d}t = -2x \, \mathrm{d}x \end{bmatrix} = \int \frac{1}{\sqrt{t}} \cdot \left(-\frac{\mathrm{d}t}{2}\right)$$

$$= -\frac{1}{2} \cdot 2\sqrt{t} + C_1 = -\sqrt{1-x^2} + C_1.$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \, \mathrm{d}x = e^x + C$$

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$

$$\int \cos x \, \mathrm{d}x = \sin x + C$$

$$\int \sin x \, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 49(d)

Izračunajte integral $\int \ln x \, \mathrm{d}x$.

Rješenje. Sami:

$$\int \ln x \, \mathrm{d}x = x \ln x - x + C.$$